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College Algebra: The Abstract and the Applied

This course combines the basic topics covered in a traditional college algebra course with Great Books readings that are integrated into the curriculum The basic topics include a brief review of intermediate algebra, equations and inequalities, functions and their graphs, exponential and logarithmic functions, and linear and nonlinear systems. There also selections from topics such as graphing of the conic section, introduction to sequences and series, permutations and combinations, the binomial theorem, and theory of equations.

The Great Books section of this course considers algebraic ideas both in today's applied world and relative to their origins. The origins of algebraic ideas and concepts will be explored via the writings of early mathematicians and philosophers. Students can expect to enrich their mathematical knowledge using handouts provided in class, web research, and other resources. Successful completion of this course ensures the student a well-rounded perspective on algebra.

Table of Contents

The contents of this module are as follows:

Theme

The theme of this course is mathematics as viewed from either an abstract/generalized or an applied perspective. Pursuing the generality of mathematics can be viewed as the pursuit of truth; pursuing the applications of mathematics can make the human race happier through our understanding of the physical universe.

In this course, we explore the idea that as concepts become more general and abstract, the number of their applications increases. Abstractness is important in mathematics, sometimes for its inherent beauty and elegance, but also because generality promotes utility.

The theme of abstract vs. applied is an important one because mathematics can be viewed from both perspectives, and often these two contradictory views are tied in ways that are unexpected (e.g., Einstein's generalizations and their implications for physics and the universe).

Techniques and Pitfalls

Techniques: I've used the following techniques to engage students in the Great Books approach:

- leading questions
- in-class discussions
- take-home questions
- asking students to find related material on the Internet
- background research (biographies, etc.)

Pitfalls: I've encountered some of the following problems while teaching this course:

- some material is over the students' heads
- there is a lot of reading in addition to traditionally difficult material (in regular curriculum)
- students may not "get it" or appreciate it
- students may be hesitant to sign up for this section

While some students have expressed their concern with the difficulty (and volume) of the readings, many students have expressed their satisfaction with the course. Students appreciate the opportunity to see the context and history of mathematics and move beyond (what some of them perceive as) the "dry" mechanics that most students experience in basic math courses.
Students become engaged in discussions about the authors and their contributions to mathematics and the manner in which humans view their world and universe.

Great Books Aspects of the Course

The Great Books enhancement of this course allows students to experience how math was originally "created" and "conceptualized." Many students see math as a set of rules and conventions that have always been in place and have very little meaning other than as repeated mechanical applications. In this course, students can see how mathematical concepts arose from physical problems that early mathematicians and scholars encountered in their ordinary lives. Students can also see how mathematical rules have been created out of scholars' desire to formalize general properties derived from both concrete observations and logical processes in their minds.

Through the Great Books readings in this course, students can see how mathematics is its own language, created and developed by early scholars such as Pythagoras and Euclid. Mathematics, in the form we now recognize, did not exist at one point. While the properties of mathematics have always been part of nature and the physical world, humans had to conceptualize how these rules were written and the interrelationships between the rules. Concepts such as the number "0" are quite complex, even though we now take this idea for granted.

The Great Books readings also show how our assumptions in mathematics cannot be taken for granted, and that changes in these assumptions, conventions and definitions have profound implications for the entire conceptualization of mathematics.

Students will also be able to see how mathematics ties in closely to other disciplines such as philosophy, religion, science, art, and logic. Many early mathematicians and scientists discussed the ties between these disciplines, including their convergence and divergence at various times in history.

Students will also see how mathematics can be both applied and generalized. Mathematics is viewed from both perspectives by the Great Books authors. Earlier mathematicians and scientists such as Kepler, Galileo, Descartes, and Newton show how physical problems can be rewritten in the common language of mathematics. Einstein shows how mathematical ideas are generalized and how the abstractness of mathematics can be used to create new conceptual models for how the universe might work.

Text Selections

The basic college algebra text used in this course is:

College Algebra: Understanding Functions: A Graphing Approach, Goodman and Hirsch (Thomson-Cole, 2004)

The following texts (or excerpts of them) by Great Books authors are also read in this course:

Euclid, *Elements*
Albert Einstein, *Relativity: The Special and General Theory; "*Gaussian Co-ordinates"
Rene Descartes, *Discourse on Method* and *Geometry*
Galileo Galilei, *Dialogues Concerning Two New Sciences*
Leonhard Euler, "Euler-McLaurin Summation Formula," "Dances between Continuous and Discrete
"Investigations on the Imaginary Roots of Equations"
Johannes Kepler, *Harmonies of the World,* book 5
Archimedes, "Archimedes' Screw"
Sir Isaac Newton, *Principia: The Mathematical Principles of Natural Philosophy*
Pythagoras (to be determined)

Sources for Great Books Texts

Author	Reading	Relates to	Where to Find
Archimedes	Archimedes' Screw	General applications	http://www.math.myu.edu/~crorres/Archimedes/Screw/SourcesScrew.html
Rene Descartes	1. *Discourse on the Method of Rightly Conducting the Reason, and Seeking Truth in the Sciences* 2. Excerpts from *Geometry*	General reasoning Equations and roots	http://www.literature.org handout
Albert Einstein	1. Gaussian Co-ordinates 2. Chapters 3, 17, and 24 from handout	Cartesian and other coordinate systems, infinity, d = rt, algebraic equations and applications	http://www.bartleby.com/173/25.html handout
Euclid	Selections from Euclid's *Elements*	Geometry, basic definitions	handout
Leonhard Euler	1. Euler-McLaurin Summation Formula 2. Dances between Continuous and Discrete 3. Investigations on the Imaginary Roots of Equations	Series (9.2) Series Polynomial functions	handout handout
Galileo	Excerpt from *Dialogues Concerning Two Sciences*	Conic sections	handout
Johannes Kepler	Excerpt from *Harmony of the World,* Book 5	Conic sections	handout
Monsieur Lame	Partitioning Polygons into Triangles	Lines and distances	handout
Isaac Newton	Excerpts from *Principia*	Conic sections	handout
Thompson and Feit letter	Correspondence between Thompson and Feit regarding "Solvability of Groups of Odd Order"	Abstract algebra (just for fun)	Copy from teacher

Importance of Texts

The importance of some of these Great Books readings is discussed below.

Euclid: Euclid provided many basic definitions for both geometry and mathematics. While many concepts are "understood," Euclid provided a solid basis for further definition and development of rules, relationships, and theorems. Students can see how his *Elements* set the style for theorems and proofs as seen in modern mathematics.

Einstein: Einstein was revolutionary in his thinking. He questioned and redefined many basic assumptions about mathematics, physics, and the universe. He used simple algebraic concepts to describe fundamental properties of the universe and showed how basic mathematical concepts can be generalized to fit into numerous paradigms. Einstein's genius is further demonstrated by the accessibility and readability of many of his works.

Descartes: Descartes studied mathematics and related disciplines from both an abstract perspective and an applied approach. He discussed the relationship between philosophy, religion, mathematics, and other disciplines and proposed a method for reasoning and "seeking truth in sciences." His work was groundbreaking in that he was the first to describe a

method of reasoning different from that of the Greek philosophers. He also worked on a specific and concrete theory of equations, based on physical phenomena such as lines, planes, and boxes. He demonstrated how mathematics describes two-, three-, and more dimensional objects.

Kepler, Galileo, Newton: These three mathematicians explored the relationship between mathematics and early physical problems, such as those in astronomy and navigation. Galileo studied astronomy and tied planetary motion to simple mathematical descriptions. Kepler studied astronomy and formulated laws of planetary motion, focusing on elliptical orbits. Newton studied the concept of gravity and conceptualized basic mathematical principles of gravity and other physics concepts. In addition, Newton, independently of and concurrently with Leibniz, conceptualized the idea of infinitesimals and laid the groundwork for our current understanding of calculus.

Structure of the Course

Students have quizzes at the beginning of each class. Approximately half of these quizzes cover the mechanics expected in a college algebra course. The other half of the quizzes cover variations of the questions listed below. Some questions are short and can be answered in class, while others are taken home for 3–7 days.

Half-hour discussions are provided every week to discuss the readings and the themes listed below.

Two writing assignments are given in the course. One is a general summary of the readings, and the other is a research paper. In the research paper, students will choose their own readings and analyze them based on a given theme (a choice is provided). I encourage students to work in groups on the research paper in order to promote discussion and deeper thought on their reading and writing.

Questions focused on the readings are also incorporated into the four exams in the course. Approximately 10–20 percent of each exam is based on material from the readings. There is also a final exam.

Special features of this course include a writing workshop conducted by a member from the English Department's writing center; and a movie, *What the Bleep Do We Know?* The movie focuses on the themes of mathematics and the universe and the relationship between the concrete and the abstract.

Syllabus

Week 1 *College Algebra*, chapter 1; pre-test, take-home quiz on chapter 1

Week 2 *College Algebra*, 2.1, 2.2
College Algebra, 2.3, 2.4
Homework: Euclidean terminology

Week 3 *College Algebra*, 2.5, 2.6; Euclid, *Elements*, pp. 1–2, 272–282
Review
Homework: definitions

Week 4 EXAM 1
College Algebra, 3.1, 3.2
Homework: Einstein, "Gaussian Co-ordinates"

Week 5 *College Algebra*, 3.3, 3.4
College Algebra, 3.5, 3.6; Descartes, *Discourse on Method*

Week 6 *College Algebra*, 3.7, 4.1
Homework: Read Einstein, *Relativity: The Special and General Theory*
College Algebra, 4.2, 4.3
Homework: Read Galileo, *Dialogues Concerning Two New Sciences*

Week 7 *College Algebra*, 4.4, 4.5; Einstein, *Relativity: The Special and General Theory*
College Algebra, 4.6; Review

Week 8 EXAM 2
Movie: *What the Bleep Do We Know?*

Week 9 *College Algebra*, 5.1–5.3
Homework: Read Euler, "Investigations on the Imaginary Roots of Equations"
College Algebra, 5.4, 5.6
Homework: Read Descartes, *Geometry*

Week 10 *College Algebra*, 6.1, 6.2
Writing workshop
College Algebra, 6.3, 6.4

Week 11 *College Algebra*, 6.5; Review
ESSAY due
EXAM 3

Week 12 *College Algebra*, 7.1, 7.3
Homework: Read Kepler, *Harmonies of the World*, book 5
College Algebra, 7.4, 7.6

Week 13 *College Algebra*, 7.8, 7.8
College Algebra, 8.1, 8.2
Homework: Read Archimedes, "Archimedes' Screw"

Week 14 *College Algebra*, 8,3, 8.4
Homework: Read Newton, *Principia: The Mathematical Principles of Natural Philosophy*
RESEARCH PAPER due
College Algebra, 8.6, 9.1
Homework: Read "Euler-McLaurin Summation Formula"

Week 15 *College Algebra*, 9.2; Review
"Euler-McLaurin Summation Formula"
EXAM 4

Week 16 FINAL EXAM

Sample Questions

The questions listed below can be used in the following ways: (1) to lead class discussions on readings and concepts; (2) as take-home quizzes (weekly, generally); and (3) as exam take-home questions.

General Questions for All Readings and Papers

1. How were concepts, now considered commonplace and routine, initially defined and conceptualized? How would you have defined these concepts?

2. What is the difference between Euclidean and non-Euclidean geometry?

3. Why is the rectangular coordinate system called the Cartesian coordinate system?

4. How do math and physics relate to each other?

5. Why was Einstein's theory of relativity so revolutionary? (Please consider relative to mathematics.)

6. How do Leibniz's and Newton's approaches to calculus compare? (optional)

7. What are the different coordinate systems and how are they used?

8. How is "i" applied in "real life"?

9. How does algebra "describe" real (e.g., physical) life?

10. Can mathematics be viewed as both abstract and applied? How so? Provide examples.

Questions for Specific Readings

Euclid

1. What ideas/concepts/definitions do we take for granted, that were carefully, thoughtfully, and deliberately described by Euclid?

2. How would you define Euclid's definitions (e.g., point, line, circle, etc.)?

3. How do Euclid's definitions relate to our current definitions?

4. Do the "common notions" look familiar? If so, how?

5. Re-create one of Euclid's elements (theorems), or translate into the modern algebraic equivalent.

Einstein

1. How does Einstein's theory of relativity relate to Euclid's geometry?

2. What is the difference between Euclidean and non-Euclidean geometry?

3. How does Einstein view infinity and unboundedness? How does this relate to your view of infinity?

4. What does infinity mean?

5. What mathematical ideas/concepts are tied to Einstein's theories? Give examples (e.g., distance = rate * time).

6. What is the meaning and significance of the equation $E = mc^2$?

7. How are coordinate systems discussed and applied in Einstein's work?

8. State the theory of relativity in your own words and describe how it relates to coordinate systems.

9. Show that x' = ct' (Lorentz transformation) algebraically, as discussed on p. 84 of Einstein's *Relativity*.

10. Show, using the relationship distance = rate * time, that seeing lightning in two places simultaneously while traveling at a constant velocity is a contradiction.

11. What is the significance of four dimensions in Einstein's work?

12. How does "i" relate to Einstein's work?

Descartes

1. What is Descartes' view of mathematics and science?

2. How do mathematics, philosophy, logic, and religion relate to each other?

3. Compare/contrast Descartes' *Discourse* to his *Geometry*. What is his purpose in each of these writings? Are there any similarities?

4. What does Descartes say about roots and equations? How does this relate to what you study in algebra today?

5. How does Descartes' work with roots and equations relate to physical phenomena (e.g., lines, solids, etc.)?

6. How is "i" applied in Descartes' work?

7. How does Descartes define basic concepts which are common knowledge to us today?

8. How does Descartes view mathematics relative to other disciplines?

9. What method(s) does Descartes suggest for reasoning and "seeking truth in the sciences"?

10. How do Descartes' examples and anecdotes add to his "discourse"?

Galileo

1. How does Galileo use mathematics?

2. How does his approach compare to other mathematicians exploring similar themes, such as Kepler and Newton?

Kepler

1. What uses does Kepler find for mathematics?

2. Is his application of mathematics similar to that of Galileo and Newton?

3. What is unique about Kepler's view of mathematics?

Newton

1. Much of Newton's work relate to physics. How does physics, as Newton describes it, relate to the mathematics we study?

2. Write/derive mathematical equations corresponding to one of Newton's concepts.

3. How do Newton's writings on planetary motion relate to conic sections?

Essay on Readings

In this paper, you will summarize, compare, and contrast at least five readings from our reading list. Your finished product should be at least three double-spaced pages long. Projects not meeting the criteria I've listed above will not be accepted.

You may select how to organize your paper, but it must contain the following:

- A summary of each reading
- An overall comparison/contrast of five different readings in the course, using a theme listed below or your own theme

Here are some ideas for organizing your paper (you don't have to choose one of these ideas):

- Compare/contrast how five or more of the authors address a similar mathematical or algebraic idea
- Track the progress of a mathematical idea or concept through time

Note: If you use references, you must cite the references. Choose a "style" that you're comfortable using (such as APA, MLA, etc.) to cite your references. If you do not cite your references, you will receive a "0" for the project.

Grading Rubric for Essay on Readings

The total possible number of points for this paper is 150 points.

Category and Points Possible	A	B	C	D
Content/Theme **20 points**	Very clear and effectively limited theme. Clear purpose and direction. 20	Clear main idea and somewhat effectively limited theme. Purpose and direction of essay somewhat clear. 17	Implicit main idea and partially limited theme. Purpose and direction discernable. 15	Unclear main idea that is insufficiently limited (too broad or too narrow). No evidence of purpose or direction. 13
Organization **25 points**	Information is very organized with well-constructed paragraphs and subheadings. Main thesis of essay is clear. 25	Information is organized with well-constructed paragraphs. Main thesis of essay is clear. 22	Information is organized, but paragraphs are not well-constructed. Main thesis of essay is unclear. 19	The information appears to be disorganized. Main thesis of essay is unclear or non-existent. 16
Relevance of Arguments **75 points**	Arguments clearly support the main theme. Several supporting details and/or examples are included. 75	Arguments clearly support the main theme. 1–2 supporting details and/or examples are provided. 64	Arguments clearly support the main theme. No details and/or examples are given. 56	Arguments have little or nothing to do with the theme of the essay. 49
Mechanics **15 points**	No grammatical, spelling, or punctuation errors. 15	Almost no grammatical, spelling, or punctuation errors, and the few that are present do not detract from the clarity of the essay. 13	A few grammatical, spelling, or punctuation errors. Errors make essay unclear in areas. 11	Many grammatical, spelling, or punctuation errors. Content of essay is unclear due to errors. 10
Basic Requirements **15 points**	Essay meets all of the following requirements: 1. Length is 3+ pages 2. Paper is typed 3. Paper is double-spaced 4. Paper is in font no larger than 12 points 5. Paper written to the appropriate audience 6. Sources are documented appropriately 7. Sources are appropriate for paper 15	Paper meets 6 requirements. 13	Paper meets 4–5 requirements. 11	Paper meets 3 or less requirements. 10

Research Paper

In this paper, you will select a paper or book by a Great Books author of your choice (from those listed below). You must choose a reading that is *not* on our reading list, and the reading should be a primary source. Alternatively, you may choose an author from our readings, as long as your chosen reading(s) are different than those assigned in class. A maximum of two class members or groups may select each author. Your source reading(s) must be at least 10 pages long (in combined length). Your finished product should be at least three double-spaced pages long. If you have an author not listed below in mind, please obtain approval from me at least three weeks before the paper is due. Projects not meeting the criteria I've listed above will not be accepted.

I encourage you to work in groups of up to three people. These ideas and works are best understood through discussion with your peers.

If you wish to present your "paper" in a non-traditional form (such as a play, video, mock interview, or something else), please feel free. Please use a form which allows you to best express your thoughts.

Your paper should include and/or address one of the following questions or themes:

- A contrast/comparison of your perceptions of the authors' writings vs. other scholars' perceptions of their writings.
- A contrast/comparison of two or more authors' ideas on an algebraic/mathematical concept. You may compare your selected reading to one that's been assigned in class.
- You may answer one of the questions listed at the bottom of our reading list. (If you choose this option, you may use one of our listed readings in addition to your own sources.)
- A mathematical derivation or construction of a concept outlined by the author.
- Some mathematical concepts to explore can include the idea of infinity, coordinate systems and their applications, roots of equations, imaginary/complex numbers, conic sections, and more.

Please note that a summary of the authors' ideas is not enough.

Your paper should include:

- An introduction and summary of your reading(s)
- A thoughtful analysis of the question/theme you've chosen.
- Please include a copy of the reading(s) at the end of your paper
- Please cite all references (including the reading[s] you attach at the end of your paper)

Great Books Authors (and Others)

- Archimedes
- Aristotle
- Bolzano
- Descartes
- Einstein
- Euclid
- Galileo
- Galois
- Kepler
- Lagrange
- Leibniz
- Newton
- Pascal
- Plato

- Poincare

You can find resources for these authors via the Internet, the campus library, in bookstores, or in major university libraries.

Note: If you use references, you must cite the references. Choose a "style" that you're comfortable using (such as APA, MLA, etc.) to cite your references. If you do not cite your references, you will receive a "0" for the project.

Grading Rubric for Research Paper

The total possible number of points for this paper is 150 points.

Category and Points Possible	A	B	C	D
Content/Theme **20 points**	Very clear and effectively limited theme. Clear purpose and direction. 20	Clear main idea and somewhat effectively limited theme. Purpose and direction of report somewhat clear. 17	Implicit main idea and partially limited theme. Purpose and direction discernable. 15	Unclear main idea that is insufficiently limited (too broad or too narrow). No evidence of purpose or direction. 13
Organization **25 points**	Information is very organized with well-constructed paragraphs and subheadings. Main thesis of report is clear. 25	Information is organized with well-constructed paragraphs. Main thesis of report is clear. 22	Information is organized, but paragraphs are not well-constructed. Main thesis of report is unclear. 19	The information appears to be disorganized. Main thesis of report is unclear or nonexistent. 16
Relevance of Arguments **45 points**	Arguments clearly support the main theme. Several supporting details and/or examples are included. 45	Arguments clearly support the main theme. 1–2 supporting details and/or examples are provided. 38	Arguments clearly support the main theme. No details and/or examples are given. 34	Arguments have little or nothing to do with the theme of the essay. 29
Quality of Sources **30 points**	Sources are appropriate to your theme and meet the criteria listed in the assignment. Your source(s) should be at least 10 pages long. 30	Sources are mostly appropriate to your theme and generally meet the criteria. Your source(s) are 10+ pages long. 26	Sources are somewhat appropriate to your theme and generally meet the criteria. Your source(s) may not be 10 pages. 23	Sources are inappropriate and don't meet the criteria. Your source(s) may not be 10 pages long. 20
Mechanics **15 points**	No grammatical, spelling, or punctuation errors. 15	Almost no grammatical, spelling, or punctuation errors, and the few that are present do not detract from the clarity of the report. 13	A few grammatical, spelling, or punctuation errors. Errors make report unclear in areas. 11	Many grammatical, spelling, or punctuation errors. Content of report is unclear due to errors. 10
Basic Requirements **15 points**	Report meets all of the following requirements: 8. Length is 3+ pages 9. Paper is typed 10. Paper is double-spaced 11. Paper is in font no larger than 12 points 12. Paper written to the appropriate audience 13. Sources are documented appropriately 14. Sources are appropriate for paper 15	Paper meets 6 requirements. 13	Paper meets 4–5 requirements. 11	Paper meets 3 or less requirements. 10